Solution to MHT CET – 2021

24th September (Shift - 1)

Section I

PHYSICS

1. (C)

Moment of inertia $I = 2m\left(\frac{d}{2}\right)^2 = \frac{md^2}{2}$

Kinetic energy $k = \frac{1}{2}I\omega^2$

$\therefore \quad \omega^2 = \frac{2k}{I} = 2k \cdot \frac{2}{md^2} = \frac{4k}{md^2}$

$\therefore \quad \omega = \frac{2}{d}\sqrt{\frac{k}{m}}$

2. (C)

Potential energy $= -\frac{1}{4\pi\varepsilon_0} \cdot \frac{ze^2}{r}$

Kinetic energy $= -\frac{1}{2}(\text{P.E.}) = \frac{1}{8\pi\varepsilon_0} \cdot \frac{ze^2}{r}$

Total energy $= \frac{1}{2}(\text{P.E.}) = -\frac{1}{8\pi\varepsilon_0}\frac{ze^2}{r}$

As r decreases K.E, increases
As r decreases P.E. and T.E. decreases (since they are negative).

3. (D)

4. (A)

In series combination, the effective spring constant

$k_1 = \frac{k}{2}$

In the parallel combination, the effective spring constant is

$k_2 = 2k$

$f_1 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}} = \frac{1}{2\pi}\sqrt{\frac{k}{2m}}$

$f_2 = \frac{1}{2\pi}\sqrt{\frac{k_2}{m}} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

$\therefore \quad \frac{f_1}{f_2} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

5. (B)

$p = \frac{1}{f} = (\mu - 1)\left(\frac{1}{R} + \frac{1}{R}\right) = (\mu - 1)\frac{2}{R}$

$p' = \frac{1}{f'} = (\mu - 1)\frac{1}{R}$

$\therefore \quad p' = \frac{p}{2}$ and $f' = 2f$

6. (C)

Momentum $p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{3 \times 10^{-9}} = 2.21 \times 10^{-25}$ kg ms^{-1}

Energy $E = \frac{hc}{\lambda} = pc = 2.21 \times 10^{-25} \times 3 \times 10^{8}$

$= 6.63 \times 10^{-17}$ J

7. (B)

Maximum height $h = \frac{u^2 \sin^2 \theta}{2g}$

For the first stone $\theta = 90°$, $\sin 90° = 1$

$\therefore \quad h_1 = \frac{u^2}{2g} = \frac{v^2}{2g}$

For the second stone $h_2 = \frac{v^2 \sin^2 30°}{2g} = \lambda$

$= \frac{v^2}{2g} \times \frac{1}{4}$

The masses are same. Hence ratio of potential energies

$\frac{p_1}{p_2} = 4$

8. (C)

In the first case θ is the critical angle

Hence $\sin\theta = \frac{1}{\mu}$

In the second case $\frac{\sin\theta}{\sin r} = \mu \qquad \therefore \quad \sin r = \frac{\sin\theta}{\mu}$

$\therefore \quad \sin r = \frac{1}{\mu^2} \qquad \therefore \quad r = \sin^{-1}\left(\frac{1}{\mu^2}\right)$

9. (B)

Reynold number is given by

$R_n = \frac{v_c \rho d}{\eta}$

$\therefore \quad v_c = \frac{R_n \eta}{\rho d}$

$R_n = 2500$, $\eta = 10^{-3}$ Ns/m^2, $R_n = 2500$, $\rho = 10^3$ kg/m^3

$d = 2r = 2$ cm $= 2 \times 10^{-2}$ m

Substituting these values and calculating we get critical velocity $v_c = 0.125$ m/s

10. (D)
When length decreases, the frequency of wire will increase. If n is the frequency of the tuning fork, then we have
$n \times 25 = (n + 6) \times 24$
$\therefore\ n = 144$ Hz

11. (C)
The capacitance of a parallel plate capacitor is given by
$$C = \frac{kA_0}{d}$$
If the distance between the plates d is decreased to half, the capacitance will become double.

12. (C)
The wavelength corresponding to 2.5 eV is given by
$$\text{Energy (eV)} = \frac{12400}{\lambda\,(\text{Å})}$$
$$\therefore\ \lambda = \frac{12400}{2.5} = 5000\ \text{Å}$$
To get detected, the signal should have energy greater than 2.5 eV or wavelength less than 5000 Å.

13. (C)
$$\lambda_1 - \lambda_2 = \frac{V}{f_1} - \frac{V}{f_2}$$
$$= \frac{320}{320} - \frac{320}{450}$$
$$= 1 - \frac{2}{3} = \frac{1}{3}\text{ m} = 0.33\text{ m} = 33\text{ cm}$$

14. (B)
For adiabatic expansion we have
$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$
$$\therefore\ \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{L_1}{L_2}\right)^{\gamma-1} \quad [\because v \propto L]$$
$$= \left(\frac{L_1}{L_2}\right)^{\frac{5}{3}-1} \quad [\text{For monoatocomic gas } \gamma = \frac{5}{3}]$$
$$= \left(\frac{L_1}{L_2}\right)^{\frac{2}{3}}$$

15. (D)
At constant pressure $W = P\,.\,\Delta V$
Heat supplied $Q = nC_v\,\Delta T = n \times \frac{3R}{2} \times \Delta T$ (For monoatomic gas $C_v = \frac{3}{2}R$)

For an ideal gas $PV = nRT$
$\therefore\ P\,\Delta V = n\,R\,\Delta T$
$\therefore\ W = Q$
$\therefore\ Q = \frac{3W}{R}$

16. (A)

The weight of the disc is balanced by the force due to the surface tension and the upthrust of water.

The component of surface tension in vertically upward direction is $T\cos\theta$ and the force acting due to it is $2\pi rT\cos\theta$.

The upthurst is equal to the weight of the water displaced (W).

$\therefore$ Weight of the disc $= 2\pi r\,T\cos\theta + W$

17. (B)

Acceleration $a = \omega^2 x$

$$\therefore\ \omega^2 = \frac{a}{x} = \frac{0.5}{0.02} = 25$$

$\therefore\ \omega = 5$ rad/s

$V_{max} = A\omega^2 = 0.1 \times 5 = 0.5$ m/s

18. (B)

The electric field intensity on the surface is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad \ldots(1)$$

But $\quad q = \frac{4}{3}\pi r^3\rho$

Putting this value of q in Eq.(1) and simplifying we get

$$E = \frac{\rho r}{3\varepsilon_0}$$

19. (C)

$$\frac{N_s}{N_p} = 20 = \frac{I_p}{I_s}$$

$$\therefore\ I_s = 20I_p = 20 \times 2 = 40 \text{ A}$$

20. (C)

Maximum wavelength of Balmer series is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R}{36} \qquad \ldots(1)$$

Maximum wavelength is given by

$$\frac{1}{\lambda'} = R\left(\frac{1}{4} - \frac{1}{\infty}\right) = \frac{R}{4} \qquad \ldots(2)$$

Dividing Eq.(2) by Eq.(1)

$$\frac{\lambda}{\lambda'} = \frac{9}{5}$$

21. (A) **22.** (C)

23. (C)
The electric potential decreases in the direction of the field. Hence field is maximum at B.

24. (B)
For output Y to be '1', the output of the OR gate should be '0'. This is possible only when both the inputs are zero.

25. (C)
Photoelectric current is proportional to the intensity of the incident light.

26. (B)
If x is the fringe width, then there will be a minimum in front of the slit if

$$\frac{d}{2} = \frac{x}{2}, \frac{3x}{2}, \frac{5x}{2}, \ldots\ldots$$

or $d = x, 3x, 5x, \ldots\ldots$

$$\therefore \quad x = d, \frac{d}{3}, \frac{d}{5}, \ldots\ldots$$

$$\lambda = \frac{xd}{D}$$

$$\therefore \quad \lambda = \frac{d^2}{D}, \frac{d^2}{3D}, \frac{d^2}{5D}$$

$\therefore$ λ is inversely proportional to D, 3D, 5D, ….

27. (C)
Distance between the two dark fringes on either side of the central bright fringe is given by

$$2x = \frac{2\lambda D}{a}$$

Putting $\lambda = 5.4 \times 10^{-7}$ m, D = 2m and $a = 0.9 \times 10^{-3}$ m
we get 2x = 2.4 mm

28. (D)
In the part of an a.c. circuit

$$V_A - V_B = L\frac{dI}{dt} + IR$$

$$0.5 = 8L + 0.5 \times 0.2$$

$$\therefore \quad 8L = 0.4$$

$$\therefore \quad L = \frac{0.4}{8} = 0.05\text{ H}$$

29. (C)
$V = 20\cos 2000\,t \quad \therefore \omega = 2000$ rad/s

$X_L = \omega L = 2000 \times 5 \times 10^{-3} = 10\,\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{2000 \times 50 \times 10^{-6}} = 10\,\Omega$$

$\therefore$ $X_L = X_C$ which gives resonance.

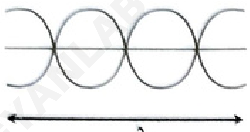
$Z = R = 10\,\Omega$

[Here we are ignoring resistance of the coil which is given as 5 Ω. Otherwise, the total resitance of the circuit will be 15 Ω]

The peak current $I_0 = \frac{V_0}{R} = \frac{20}{10} = 2\text{ A}$

R.M. S. current $I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}\text{A}$

30. (B)



$L = 3\frac{\lambda}{2}$

Since $L = 3\frac{\lambda}{2}$, it is third harmonic or second overtone.

31. (B)

In circuit (a) the two diodes are in series and one of the diodes is reverse biased. Hence no current will flow in the circuit and ammeter will not show any deflection.

32. (C)

$$\left(\frac{R_1}{R_2}\right)^3 = \left(\frac{T_1}{T_2}\right)^2 = (8)^2 = 64$$

$$\therefore \quad \frac{R_1}{R_2} = (64)^{\frac{1}{3}} = 4$$

$$\therefore \quad R_1 = 4R_2$$

$$\therefore \quad R_1 - R_2 = 4R_2 - R_2 = 3R_2$$

33. (A)

$Q_1 = CV_1$, $Q_2 = CV_2$, $Q = Q_1 + Q_2 = 2CV$

$$\therefore \quad CV_1 + CV_2 = 2CV$$

$$\therefore \quad V = \frac{V_1 + V_2}{2}$$

$$\text{Decrease in energy} = \frac{1}{2}CV_1^2 + \frac{1}{2}CV_2^2 - \frac{1}{2}(2C)\left(\frac{V_1 + V_2}{2}\right)^2$$

$$= \frac{1}{2}C\left[V_1^2 + V_2^2 - 2\left(\frac{V_1 + V_2}{2}\right)^2\right]$$

$$= \frac{1}{2}C\left[V_1^2 + V_2^2 - \frac{1}{2}(V_1 + V_2)^2\right]$$

$$= \frac{1}{4}C\left[2V_1^2 + 2V_2^2 - (V_1 + V_2)^2\right]$$

$$= \frac{1}{4}C(V_1 - V_2)^2$$

34. (C)

W = 49 N

$$\therefore \quad m = \frac{W}{g} = \frac{49}{9.8} = 5\text{ kg}$$

$$W' = W - ma = 49 - 5 \times 5 = 49 - 25$$
$$= 24\text{ N}$$

35. (C)

$A + B + C = 420$

$A + B = 220$ cm $\quad \therefore \; C = 420 - 220 = 200$ cm

$B + C = 320$ cm $\quad \therefore \; B = 320 - 200 = 120$ cm

$A = 220 - B = 100$ cm

$\therefore \; A : B : C :: 1 : 1.2 : 2$

36. (D)

$$L = 2\pi r \qquad \therefore \; r = \frac{L}{2\pi}$$

Area of the loop $A = \pi r^2 = \pi \frac{L^2}{4\pi^2} = \frac{L^2}{4\pi}$

Magnetic moment $M = IA = \frac{IL^2}{4\pi}$

37. (C)

Force on each conductor is given by

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2I_1I_2}{d} \cdot \ell$$

The force will be attractive. If the direction of current is reversed in one conductor the force will become repulsive.

$$\therefore \; F' = -\frac{\mu_0}{4\pi} \cdot \frac{2I_1I_2}{3d} \cdot \ell \quad = -\frac{2}{3}F$$

38. (C)

When string is horizontal its speed is given by

$$V = \sqrt{3gr}$$

Centripetal acceleration $= \frac{V^2}{r} = 3g$

39. (D)

Induced emf $e = \frac{-N(\phi_2 - \phi_1)}{t} = \frac{-NBA(\cos\theta_2 - \cos\theta_1)}{t}$

Assuming that the coil was initially placed perpendicular to the magnetic flux $\theta_1 = 0°$ and $\theta_2 = 90°$.

$$e = \frac{-NBA(\cos 90° - \cos 0°)}{t}$$

$$= \frac{-NBA(0-1)}{t} = \frac{NBA}{t}$$

$$= \frac{600 \times 5 \times 10^{-5} \times 0.06}{0.2} = 9 \times 10^{-3}\text{ V}$$

40. (B)

If L is the inductance of the coil, then energy stored is given by

$$W=\frac{1}{2}LI^2 \quad \text{but} \quad L=\frac{N\phi}{I}$$

$$\therefore \quad W=\frac{N\phi I}{2}$$

41. (B)

For isothermal expansion we have

$$P_1V_1=P_2V_2$$

$$\therefore \quad P_2=P_1\frac{V_1}{V_2}=P_1\times\frac{1}{2}=\frac{P}{2}$$

$$\therefore \quad P_i=\frac{P}{2}$$

For adiabatic process :

$$P_1V_1^\gamma=P_2V_2^\gamma$$

$$\therefore \quad P_2=P_1\left(\frac{V_1}{V_2}\right)^\gamma=P_1\left(\frac{1}{2}\right)^\gamma=\frac{P}{2^\gamma}$$

$$\therefore \quad P_a=\frac{P}{2^\gamma}$$

$$\therefore \quad \frac{P_i}{P_a}=\frac{2^\gamma}{2}=2^{\gamma-1}$$

42. (A)

43. (A)

The average translational kinetic energy of a gas molecule is given by

$$\frac{3}{2}kT=\frac{3}{2}\frac{R}{N}T$$

where N is the Avagadro number

The kinetic energy of a electrons accelerated by a p.d. of V volts is given by eV

$$\therefore \quad \frac{3}{2}\frac{RT}{N}=eV$$

$$\therefore \quad T=\frac{2eVN}{3R}$$

44. (A)

$$\frac{Q}{At}=\frac{k\Delta\theta}{d}$$

$$\therefore \quad 10=k\times\frac{9}{1.8\times10^{-2}}$$

$$\therefore \quad k=\frac{18\times10^{-2}}{9}=2\times10^{-2}\ \text{kcal/mS°C}$$

45. (B)

$$n_1\frac{5}{2}RT=n_2\frac{3}{2}R(2T)$$

$$\therefore \quad \frac{n_1}{n_2}=\frac{6}{5}$$

46. (D)

$L = I\omega = mk^2\omega$

$\therefore \ \omega = \dfrac{L}{mk^2}$

47. (A)

$I_A = I + 4I + 2\sqrt{I}\cdot\sqrt{4I}\cdot\cos\dfrac{\pi}{2}$

$= 5I$

$I_B = I + 4I + 2\sqrt{I}\cdot\sqrt{4I}\cdot\cos\pi$

$= I$

$\therefore \ I_A - I_B = 5I - I = 4I$

48. (D)

Potential energy is a half of the total energy

$\therefore \ \frac{1}{2}m\omega^2x^2 = \frac{1}{2}\left[\frac{1}{2}m\omega^2A^2\right]$

$\therefore \ x^2 = \dfrac{A^2}{2} \ \text{ or } \ x = \dfrac{A}{\sqrt{2}}$

Maximum force $F_m = \omega^2 A$

Force at a distance x is $F' = \omega^2 x$

$\therefore \ \dfrac{F'}{F_m} = \dfrac{x}{A} = \dfrac{1}{\sqrt{2}}$

$\therefore \ F' = \dfrac{F_m}{\sqrt{2}} = \dfrac{50}{\sqrt{2}} = 25\sqrt{2}$

49. (D)

$e = e_0 \cos\omega t = 10\cos 2\pi f t$

$f = 50\text{ Hz}, \ t = \dfrac{1}{600}\text{ s}$

$\therefore \ e = 10\cos 100\pi \times \dfrac{1}{600} = 10\cos\dfrac{\pi}{6}$

$= 10\dfrac{\sqrt{3}}{2} = 5\sqrt{3}\text{ V}$

50. (B)

$V = \sqrt{\dfrac{k}{\rho}}$

CHEMISTRY

51. (C)

Stability order for given divalent metal ions is :

$Cu^{2+} > Ni^{2+} > Fe^{2+} > Cd^{2+}$

The charge to size ratio of Cd^{2+} is lower and hence it forms lowest stability complex if the ligand remains same.

52. (B)

53. (C)

$R_{solution} = 30\,\Omega,\ k = 1.2\ S\ m^{-1}$

Cell constant $= k \times R_{solution}$

$= 1.2\ S\ m^{-1} \times 30\,\Omega$

$= 36\ m^{-1} = 0.36\ cm^{-1}$

54. (D)

55. (D)

Percent dissociation $= \propto \times 100$

$= 1.34 \times 10^{-2} \times 100$

$= 1.34$

56. (B)

$22400\ cm^3$ of a gas at STP $= 1$ mole of gas $= 6.022 \times 10^{23}$ molecules

57. (B)

$W_2 = 0.822\ g,\ V = 300\ mL = 0.3\ L,\ T = 300\ K,$

$M_2 = 340\ g\ mol^{-1},\ R = 0.0821\ L\ atm\ K^{-1}\ mol^{-1},\ \pi = ?$

$$\pi = \frac{W_2 RT}{M_2 V}$$

$$= \frac{0.822\,g \times 0.0821\,L\,atm\,K^{-1}\,mol^{-1} \times 300\,K}{340\ g\,mol^{-1} \times 0.3\,L}$$

$= 0.2$ atm

58. (B)

$pOH = 14 - pH = 14 - 10.9 = 3.1$

$pOH = -\log_{10}[OH^-]$

$\therefore \log_{10}[OH^-] = -pOH$

$= -3.1$

$= -3 - 0.1 - 1 + 1$

$= \bar{4} + 0.9 = \bar{4}.9$

$\therefore [OH^-] = \text{antilog}(\bar{4}.9) = 7.943 \times 10^{-4}$

For monoacidic base,

$$BOH_{(aq)} \rightleftharpoons B^+_{(aq)} + OH^-_{(aq)}$$

$[OH^-] = \propto c$

$\therefore \quad \alpha = \frac{[OH^-]}{c}$

$= \frac{7.943 \times 10^{-4}}{0.02} = 3.97 \times 10^{-2}$

Percent dissociation $= \alpha \times 100$
$= 3.97 \times 10^{-2} \times 100$
$= 3.97$

59. (C)

$H_2C-CH=CH-CH_2-OH$ → Allylic alcohol
Crotonyl alcohol

60. (C) **61.** (C)

62. (B)

Lysine is basic ∝–amino acid while others are neutral ∝-amino acids.

63. (A)

$P_{ext} = 2.5$ bar, $V_1 = x$ L, $V_2 = 4.5$ L
$W = -5\ dm^3$ bar $= -5$ L bar

$W = -P_{ext}(V_2 - V_1)$

$\therefore\ -5$ L bar $= -2.5$ bar $(4.5 - x)$ L

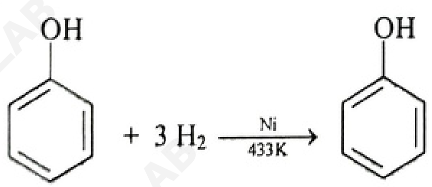
$\therefore\ 5$ L bar $= 11.25$ L bar $- 2.5$ x L bar

$\therefore\ 2.5$ x L bar $= (11.25 - 5)$ L bar

$\therefore\ x = \frac{6.25}{2.5} = 2.5$

64. (B)

65. (D)



Phenol Cyclohexanol

66. (D)

$a = 400$ pm $= 4 \times 10^{-8}$ cm
$\rho = 4\ g\ cm^{-3}$, For BCC structure, n = 2
M = ?

$$M = \frac{\rho \times a^3 \times N_A}{n}$$

$$= \frac{4\,g\,cm^{-3} \times (4 \times 10^{-8})^3\,cm^3 \times 6.022 \times 10^{23}\,\text{atoms mol}^{-1}}{2\ \text{atoms}}$$

$$= \frac{4 \times 64 \times 10^{-24} \times 6.022 \times 10^{23}}{2}$$

$= 77.08\ g\ mol^{-1}$

67. (D) **68.** (A) **69.** (A)

70. (A)

–COOR show – R effect while other groups show +R effect.

71. (C)

72. (C)

Acidic strength of halogen acids increases in the order :

HF < HCl < HBr < HI

73. (B)

$$H_2O_{2\,(g)} \longrightarrow H_2O_{(l)} + \frac{1}{2}O_{2(g)}$$

It is first order reaction.

$\therefore$ r $= k\,[H_2O_2]$

74. (C)

Ions of A are present at the 8 corners of cell.

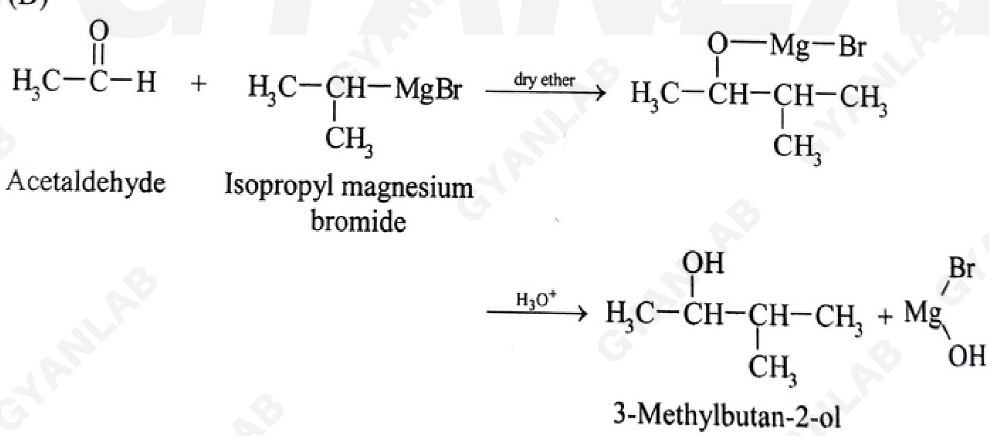
$\therefore$ Number of ions, A $= 8 \times \frac{1}{8} = 1$

Ions of B are present at the centre of 6 faces.

$\therefore$ Number of ions, B $= 6 \times \frac{1}{2} = 3$

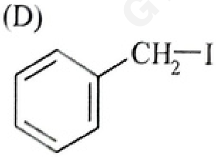
$\therefore$ Formula of the compound $= AB_3$

75. (D)



76. (C)

77. (D)

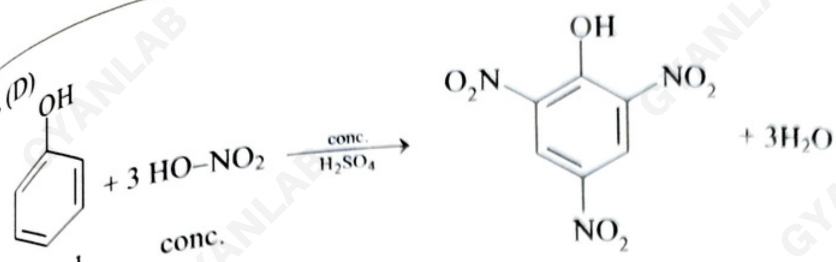


→ Benzylic halide

Iodophenylmethane

78. (D)

79. (D)



Phenol

2,4,6-Trinitrophenol
(Picric acid)

80. (A)

81. (B)

$r = k[A][B]^2$

If concentration of B is increases 3 times,

$r' = k[A][3B]^2$

$= 9k[A][B]^2$

$\therefore r' = 9r$

82. (D)

$H_3C-\underset{\displaystyle NH_2}{\underset{|}{CH}}-CH_3$

Isopropyl amine (1° amine)

83. (A)

$Na_4[Fe(CN)_6] \longrightarrow 4Na^+ + [Fe(CN)_6]^{4-}$

It is an anionic complex in which oxidation state of Fe is +2 and coordination number of Fe is 6.

84. (C)

$\overset{\overset{+5}{\uparrow}}{N}O_3^- \longrightarrow \overset{\overset{+4}{\uparrow}}{N}O_2$

85. (B)

86. (D)

$n_2 = 0.1$ mol, $P_1^0 = 24$ mm Hg

$W_1 = 1.8 \times 10^{-2}$ kg, $H_2O = 18$ g, $M_1 = 18$ g mol^{-1}

$$\therefore n_1 = \frac{W_1}{M_1} = \frac{18\,g}{18\,g\,mol^{-1}} = 1 \text{ mol}$$

$$\text{Now, } \frac{P_1^0 - P_1}{P_1^0} = \frac{n_2}{n_1 + n_2}$$

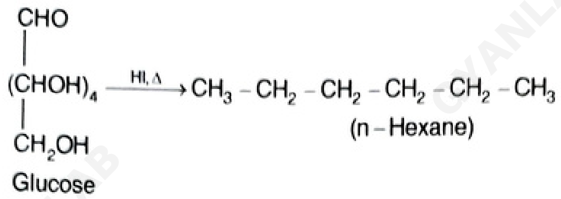
$$\therefore \frac{24\,mm\,Hg - P_1}{24\,mm\,Hg} = \frac{0.1}{1+0.1} = \frac{0.1}{1.1} = 0.09$$

$\therefore$ 24 mm Hg $- P_1 = 2.16$ mm Hg

$\therefore$ $P_1 = 24$ mm Hg $- 2.16$ mm Hg $= 21.84$ mm Hg

87. (A)

On prolonged heating with HI, it forms n–hexane, suggesting that all the six carbon atoms are linked in a straight chain.



88. (C)

89. (C)

$T = 300\ K,\ P = 5\ atm,\ R = 0.0821\ L\ atm\ K^{-1}\ mol^{-1}$

$W = 22\ g\ of\ CO_2,\ M = 44\ g\ mol^{-1},\ n_{CO_2} = \frac{22}{44} = 0.5\ mol$

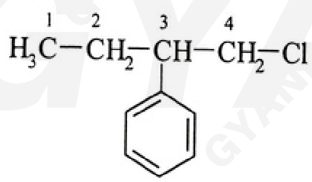
$$PV = nRT$$

$$\therefore\ V = \frac{nRT}{P}$$

$$= \frac{0.5\,mol \times 0.0821\,L\,atm\,K^{-1}\,mol^{-1} \times 300\,K}{5\,atm}$$

$$= 2.46\ L \qquad = 2.46\ dm^3$$

90. (B)



1-Chloro-2-phenylbutane (1° alkyl halide)

91. (B)

92. (D)

$W = -238\ J,\ Q = 54\ J$

According to first law of thermodynamics,

$$\Delta U = Q + W$$

$$= 54 - 238 \qquad = -184\ J$$

93. (A)

$Q = +x\ kJ\ = 1000\ x\ J,\ W = +y\ J$

According to first law of thermodynamics,

$$\Delta U = Q + W$$

$$= (1000\ x + y)\ J$$

94. (B)

$$H_3C-\underset{\underset{Cl}{|}}{CH}-CH_2-CH_3 \xrightarrow[\text{(Saytzeff rule)}]{\text{alc. KOH},\ \Delta} H_3C-CH=CH-CH_3 + KCl + H_2O$$

2-Chlorobutane $\qquad\qquad\qquad\qquad$ But-2-ene

95. (B)

96. (B)

$$A + B \longrightarrow \text{Products}$$

$$r = k[A]^2[B]^2$$

$$\therefore\ k = \frac{r}{[A]^2[B]^2}$$

$$= \frac{3.6\times10^{-2}\ \text{mol dm}^{-3}\ \text{sec}^{-1}}{[0.2]^2\ \text{mol}^2\ \text{dm}^{-6}\times[0.1]^2\ \text{mol}^2\ \text{dm}^{-6}}$$

$$= \frac{3.6\times10^{-2}}{0.04\times0.01}\ \text{mol}^{-3}\ \text{dm}^{9}\ \text{sec}^{-1}$$

$$= 90\ \text{mol}^{-3}\ \text{dm}^{9}\ \text{sec}^{-1}$$

97. (B)

$$HCl_{(aq)} + H_2O_{(l)} \rightleftharpoons H_3O^+_{(aq)} + Cl^-_{(aq)}$$

$\text{Acid}_1 \qquad \text{Base}_2 \qquad \text{Acid}_2 \qquad \text{Base}_1$

Conjugate acid-base pair$_2$

Conjugate acid-base pair$_1$

98. (D)

$$c = 0.04\ \text{mol L}^{-1},\quad k = 0.0112\ \Omega^{-1}\ \text{cm}^{-1}$$

$$\wedge = \frac{1000\,k}{c}$$

$$= \frac{1000\ \text{cm}^3\ \text{L}^{-1}\times0.0112\ \Omega^{-1}\ \text{cm}^{-1}}{0.04\ \text{mol L}^{-1}}$$

$$= 280\ \Omega^{-1}\ \text{cm}^2\ \text{mol}^{-1}$$

99. (B)

100. (C)

Section II

MATHEMATICS

101.(A)

Let $I = \int_0^4 x[x]\,dx$

$$\therefore I = \int_0^1 (0)\,dx + \int_1^2 x\,dx + \int_2^3 2x\,dx + \int_3^4 3x\,dx$$

$$= \left[\frac{x^2}{2}\right]_1^2 + \left[\frac{2x^2}{2}\right]_2^3 + \left[\frac{3x^2}{2}\right]_3^4 = \frac{1}{2}(4-1) + (9-4) + \left(\frac{3}{2}\right)(16-9) = \frac{3}{2} + 5 + \frac{21}{2}$$

$= 17$

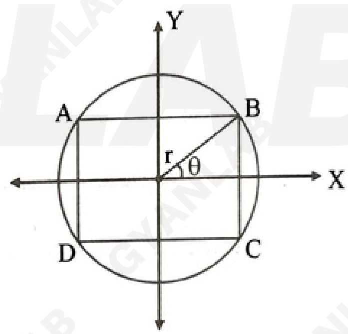
102.(B)

Let $\lim_{x\to 1} \frac{ab^x - a^x b}{x^2 - 1} = L$ ...(say)

$\therefore L = \frac{ab - ab}{1-1} = \frac{0}{0}$ form $\Rightarrow L = \lim_{x\to 1} \frac{(ab^x \log b) - (a^x \log a \cdot b)}{2x}$ ...[L′ Hospital Rule]

$$= \frac{ab\log b - ab\log a}{2} = \frac{ab}{2}\log\left(\frac{b}{a}\right)$$

103.(A)



Refer figure

B $= (r\cos\theta, r\sin\theta)$

$\therefore$ AB $= 2r\cos\theta$ and BC $= 2r\sin\theta$

A(□ABCD) = AB × BC

$\therefore f(\theta) = (2r\cos\theta)(2r\sin\theta) = 2r^2\sin 2\theta$

$f'(\theta) = 4r^2\cos 2\theta$ and when $f'(\theta) = 0$, we write

$\cos 2\theta = 0 \Rightarrow \cos 2\theta = \cos\frac{\pi}{2} \quad \Rightarrow \theta = \frac{\pi}{4}$

$f''(\theta) = -8r^2\sin 2\theta$ and $f''(\theta)\Big[_{\theta=\frac{\pi}{4}}\Big] = -8r^2 < 0$

Thus area of required rectangle is maximum when $\theta = \frac{\pi}{4}$.

$\therefore AB = 2r\cos\frac{\pi}{4} = 2r\left(\frac{1}{\sqrt{2}}\right) = \sqrt{2}\,r$ and $BC = 2r\sin\frac{\pi}{4} = 2r\left(\frac{1}{\sqrt{2}}\right) = \sqrt{2}\,r$

$\therefore$ A(□ABCD) $= (\sqrt{2}\,r)(\sqrt{2}\,r) = 2r^2$

104.(D)

The maximum value of ${}^6C_r$ occurs at $r = \frac{6}{2} = 3$

$\therefore {}^6C_3 = \frac{6!}{3!3!} = \frac{6\times5\times4}{6} = 20$

As per data given, $\left|{}^6C_3 - {}^nC_3\right| = 16$

$\therefore {}^nC_3 = 20 + 16$ or ${}^nC_3 = 20 - 16$

If $^nC_3 = 36 \Rightarrow \dfrac{n!}{3!(n-3)!} = 36 \Rightarrow n(n-1)(n-2) = 216$ is not possible for $n \in N$

If $^nC_3 = 4 \Rightarrow \dfrac{n!}{(n-3)!3!} = 4 \Rightarrow n(n-1)(n-2) = 24$

Here $24 = 2 \times 3 \times 4 \quad \Rightarrow n = 4$

105.(B)

$(fog)(x) = f\left[\dfrac{x}{x+1}\right]$

$$= \frac{\left(\frac{x}{x+1}\right)}{2\left(\frac{x}{x+1}\right)+1} = \frac{x}{x+1} \times \frac{x+1}{2x+x+1} = \frac{x}{3x+1}$$

106.(D)

Let the initial population be P and rate of increase is 8% per year.

$\therefore \dfrac{dP}{dt} = \dfrac{8}{100}P$

$\therefore \int \dfrac{dP}{P} = \int 0.08\, dt$

$\therefore \log|P| = 0.08\, t + c$

When t = 0, we get c = log P

$\therefore \log P = 0.08\, t + \log P$

When 'P' doubles, we write

$\log 2p = 0.08\, t + \log P$

$\therefore \log\left(\dfrac{2P}{P}\right) = \log 2 = 0.6912 = 0.08\, t$

$\therefore t = \dfrac{0.6912}{0.08} = 8.64$ years

107.(D)

$\tan\alpha$ and $\tan\beta$ are roots of the $ax^2 - bxy - y^2 = 0$

$\therefore \tan\alpha + \tan\beta = \dfrac{-(-b)}{-1} = -b$ and $\tan\alpha\tan\beta = \dfrac{a}{(-1)} = -a$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{-b}{1-(-a)} = \frac{-b}{1+a}$$

108.(B)

$X = x_i$	−2	−1	0	1	2
$P(X = x_i)$	0.2	0.3	0.15	0.25	0.1

$\therefore F(0) = 1 - [P(x=1) + P(x=2)]$

$= 1 - P(X > 0)$

109.(B)

$$\frac{dy}{dx} = \frac{x+y+1}{x+y-1}$$

Put $x+y=v \Rightarrow 1+\frac{dy}{dx}=\frac{dv}{dx}$

$$\therefore \frac{dv}{dx}-1=\frac{v+1}{v-1} \Rightarrow \frac{dv}{dx}=\frac{v+1}{v-1}+1 = \frac{2v}{v-1}$$

$$\therefore \int\frac{(v-1)dv}{2v} = \int dx$$

$$\therefore \int\frac{1}{2}dv-\int\frac{1}{2v}dv = \int dx \Rightarrow \frac{v}{2}-\frac{1}{2}\log|v| = x+c$$

$$\therefore \frac{x+y}{2}-\frac{1}{2}\log|x+y| = x+c$$

We have $x=\frac{2}{3}, y=\frac{1}{3}$, we get

$$\therefore \frac{1}{2}-\frac{1}{2}\log|1| = \frac{2}{3}+c \Rightarrow c=\frac{1}{2}-\frac{2}{3}=\frac{-1}{6}$$

$$\therefore \frac{x+y}{2}-\frac{1}{2}\log|x+y| = x-\frac{1}{6}$$

Multiplying both sides by 2, we get

$$\therefore (x+y)-\log|x+y| = 2x-\frac{2}{6} \Rightarrow y-x+\frac{1}{3} = \log|x+y|$$

110.(B)

$a\sin\theta = b\cos\theta \Rightarrow \tan\theta=\frac{b}{a}$

$a\cos 2\theta + b\sin 2\theta$

$$= a\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)+b\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$= a\left[\frac{1-\left(\frac{b^2}{a^2}\right)}{1+\left(\frac{b^2}{a^2}\right)}\right]+b\left[\frac{2\left(\frac{b}{a}\right)}{1+\left(\frac{b^2}{a^2}\right)}\right] = a\left(\frac{a^2-b^2}{a^2+b^2}\right)+b\left(\frac{2b}{a}\times\frac{a^2}{a^2+b^2}\right)$$

$$= \frac{a(a^2-b^2)}{a^2+b^2}+\frac{b(2ab)}{a^2+b^2} = \frac{a^3-ab^2+2ab^2}{a^2+b^2} = \frac{a^3+ab^2}{a^2+b^2} = \frac{a(a^2+b^2)}{a^2+b^2} = a$$

111.(D)

$\bar{a} = x\bar{b}+y\bar{c}$

$$\therefore 4\hat{i}+13\hat{j}-18\hat{k} = (\hat{i}-2\hat{j}+3\hat{k})(x)+(2\hat{i}+3\hat{j}-4\hat{k})(y)$$

$$= (x+2y)\hat{i}+(-2x+3y)\hat{j}+(3x-4y)\hat{k}$$

$\therefore x+2y=4, \quad -2x+3y=13 \quad$ and $\quad 3x-4y=-18$

Solving, we get $x=-2, y=3 \Rightarrow x+y=1$

112.(D)

Two intersection lines are

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = \lambda \text{ and } \frac{x-2}{1} = \frac{y+m}{2} = \frac{z-2}{1} = \mu$$

$\therefore\ 2\lambda + 1 = \mu + 2$ ...(1)

$4\lambda + 1 = \mu + 2$ ...(2)

$3\lambda - 1 = 2\mu - m$ ...(3)

From (1) and (2), we infer that $\lambda = 0$ and $\mu = -1$.

Then from (3), we get $m = -1$

113.(A)

Let $P \equiv (2, -1, 5)$ and co-ordinates of any point on the given line be

$Q \equiv (10\lambda + 11, -4\lambda - 2, -11\lambda - 8)$

d.r. of PQ are $(10\lambda + 9, -4\lambda - 1, -11\lambda - 13)$

d.r. of given line are $(10, -4, -11)$.

$\therefore\ (10\lambda + 9)(10) + (-4\lambda - 1)(-4) + (-11\lambda - 13)(-11) = 0$

$\therefore\ 100\lambda + 90 + 16\lambda + 4 + 121\lambda + 143 = 0 \Rightarrow \lambda = -1$

$\therefore\ Q \equiv (1, 2, 3)$ and $d(PQ) = \sqrt{1^2 + 3^2 + 2^2} = \sqrt{14}$ units

114.(D)

$u = \cos^3 x$ and $v = \sin^3 x$

$$\therefore\ \frac{du}{dx} = 3\cos^2 x(-\sin x) \quad \text{and} \quad \frac{dv}{dx} = 3\sin^2 x(\cos x)$$

$$\therefore\ \frac{dv}{du} = \frac{3\sin^2 x \cos x}{-3\sin x \cos^2 x} = -\tan x$$

$$\therefore\ \left(\frac{dv}{du}\right)_{x=\frac{\pi}{4}} = -\tan\left(\frac{\pi}{4}\right) = -1$$

115.(B)

We know that $\text{C.V.} = \frac{\text{Standard Deviation}}{\text{Mean}}$

$$\therefore\ (\text{C.V.})_A = \frac{12}{80} = 0.15 \quad \text{and} \quad (\text{C.V.})_B = \frac{6}{75} = 0.08$$

$$(\text{C.V.})_C = \frac{8}{70} = 0.11 \quad \text{and} \quad (\text{C.V.})_D = \frac{10}{72} = 0.14$$

C.V. is the least for division B.

116.(A)

We have $\theta = 45°$ and $m_1 = \frac{1}{2}$.

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\therefore\ \tan 45° = \left|\frac{\frac{1}{2} - m_2}{1 + \left(\frac{1}{2}\right)m_2}\right| \Rightarrow \left|\frac{1 - 2m_2}{2 + m_2}\right| = \pm 1$$

$\therefore\ 1 - 2m_2 = 2 + m_2$ or $1 - 2m_2 = -2 - m_2$ $\therefore\ m_2 = \frac{-1}{3}$ or $m_2 = 3$

117.(C)

$f(x) = \log|\sin x|$, where $x \in (0, \pi)$

$\therefore\ f'(x) = \frac{1}{\sin x} \times \cos x = \cot x$

When $f'(x) > 0$, we say $\frac{\cos x}{\sin x} > 0$

Here $\sin x > 0$ ... $[x \in (0, \pi)]$

Thus for the function to be strictly increasing, $\cos x > 0 \Rightarrow x \in \left(0, \frac{\pi}{2}\right)$ only.

118.(A)

$v = 6t - \frac{t^2}{6}$ and we know that $v = \frac{ds}{dt}$

$\therefore\ \int ds = \int\left(6t - \frac{t^2}{6}\right)dt$

$\therefore\ s = \frac{6t^2}{2} - \frac{t^3}{6(3)} + c \Rightarrow s = 3t^2 - \frac{t^3}{18} + c$

We known that s = 0, when t = 0 $\Rightarrow c = 0$

$\therefore\ s = 3t^2 - \frac{t^3}{18} \Rightarrow (s)_{t=3} = 3(3)^2 - \frac{(3)^3}{18} = \frac{51}{2}$ units

119.(A)

$y = a\cos x + b\sin x + c\,e^{-x}$

Since the equation has 3 arbitrary constants, the order of differential equation is 3.

120.(A)

Let $\sqrt{-5-12i} = a + ib$ ...$[a, b \in R]$

$\therefore\ (a + ib)^2 = -5 - 12i$

$\therefore\ (a^2 - b^2) + i(2ab) = -5 - i\,(12)$

$\therefore\ a^2 - b^2 = -5$ and $2ab = -12 \Rightarrow ab = -6 \Rightarrow b = \frac{-6}{a}$

$\therefore\ a^2 - \left(\frac{-6}{a}\right)^2 = -5$

$\therefore\ a^4 - 36 = -5a^2 \Rightarrow a^4 + 5a^2 - 36 = 0$

$\therefore\ (a^2 + 9)(a^2 - 4) = 0 \Rightarrow a^2 = 4$ ...$[a \in R]$

$\therefore\ a = \pm 2 \Rightarrow b = \frac{-6}{\pm 2} = \mp 3$

$\therefore\ \sqrt{-5-12i} = \pm(2 - 3i)$

121.(A)

Let $\bar{b} = x\hat{i} + y\hat{j} + z\hat{k}$ and we have $\bar{a} \cdot \bar{b} = 1$

$\therefore\ (\hat{i} + \hat{j} + \hat{k})\,(x\hat{i} + y\hat{j} + z\hat{k}) = 1 \Rightarrow x + y + z = 1$

$\bar{a} \times \bar{b} = \bar{c} \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = \hat{j} - \hat{k}$

$\therefore\ (z-y)\hat{i} - (z-x)\hat{j} + (y-x)\hat{k} = \hat{j} - \hat{k}$

$\therefore\ z - y = 0,\ x - z = 1,\ y - x = -1$

$\therefore\ y = z,\ z = x - 1,\ y = x - 1$

Thus $\bar{b} = x\hat{i} + (x-1)\hat{j} + (x-1)\hat{k}$

We have $x + y + z = 1$

$\therefore\ x + (x-1) + (x-1) = 1 \Rightarrow x = 1,\ y = 0,\ z = 0$

$\therefore\ \bar{b} = \hat{i}$

122.(B)

Required plane passes through (1, 2, 3) and is parallel to plne $2x + 3y - 4z = 0$. Hence equation of required plane is

$2(x-1) + 3(y-2) - 4(z-3) = 0 \Rightarrow 2x + 3y - 4z + 4 = 0$

123.(B)

$\sim[p \wedge (q \rightarrow r)]$

$\equiv \sim[p \wedge (\sim q \vee r)]$

$\equiv (\sim p) \vee \sim(\sim q \vee r)$

$\equiv \sim p \vee (q \wedge \sim r)$

124.(B)

$$(2y-1)\,dx - (2x+3)\,dy = 0 \Rightarrow \frac{dy}{dx} = \frac{(2y-1)}{(2x+3)}$$

$$\therefore \int \frac{dy}{2y-1} = \int \frac{dx}{2x+3}$$

$$\frac{\log|2y-1|}{2} = \frac{\log|2x+3|}{2} + \log c_1 \Rightarrow \log\left|\frac{2x+3}{2y-1}\right| = -\log c_1 = \log c$$

$$\therefore \frac{2x+3}{2y-1} = c$$

125.(D)

$$y = \frac{\log_e x}{\log_e 10} + \frac{\log_e 10}{\log_e x} + 1 + 1$$

$$\therefore \frac{dy}{dx} = \left(\frac{1}{\log_e 10}\right)\left(\frac{1}{x}\right) + (\log_e 10)\left[\frac{-\frac{1}{x}}{(\log_e x)^2}\right]$$

$$= \frac{1}{x\log_e 10} - \frac{\log_e 10}{x(\log_e x)^2}$$

126.(C)

Let $\bar{a} = x\bar{b}$

$\therefore\ 2\hat{i} + p\hat{j} + 4\hat{k} = 6x\hat{i} - 9x\hat{j} + q\,x\hat{j}$

$\therefore\ 2 = 6x \Rightarrow x = \frac{1}{3}$

$p = -9x \Rightarrow p = (-9)\left(\frac{1}{3}\right) = -3$ and $4 = qx = q\left(\frac{1}{3}\right) \Rightarrow q = 4(3) = 12$

127.(D)

$$y = \tan^{-1}\left[\frac{\log\left(\frac{e}{x^2}\right)}{\log(ex^2)}\right] + \tan^{-1}\left[\frac{3+2\log x}{1-6\log x}\right]$$

$$= \tan^{-1}\left[\frac{\log e - \log x^2}{\log e + \log x^2}\right] + \tan^{-1}3 + \tan^{-1}(2\log x)$$

$$= \tan^{-1}\left[\frac{1-\log x^2}{1+\log x^2}\right] + \tan^{-1}3 + \tan^{-1}\left(\log x^2\right)$$

$$= \tan^{-1}(1) - \tan^{-1}(\log x^2) + \tan^{-1}3 + \tan^{-1}(\log x^2)$$

$$= \tan^{-1}(1) + \tan^{-1}(3)$$

$$\therefore \frac{dy}{dx} = 0 \Rightarrow \frac{d^2y}{dx^2} = 0$$

128.(D)

$$\lim_{x\to1^-} f(x) = \lim_{x\to1^-} 3ax + b = 3a + b$$

$$\lim_{x\to1^+} f(x) = \lim_{x\to1^+} 5ax - 2b = 5a - 2b$$

$f(1) = 11$ and function is continuous at $x = 1$, so we write

$3a + b = 11 = 5a - 2b \Rightarrow 2a = 3b$

This condition is satisfied when $a = 3, b = 2$

129.(B)

$$P\left(\frac{A}{B}\right) \times P\left(\frac{B}{A}\right) = \frac{P(A\cap B)}{P(B)} \times \frac{P(B\cap A)}{P(A)}$$

We know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{3}{5} = \frac{3}{10} + \frac{2}{5} - P(A\cap B) \Rightarrow P(A\cap B) = \frac{1}{10}$$

$$\therefore \text{ Given Expression} = \frac{\left(\frac{1}{10}\right)}{\left(\frac{2}{5}\right)} \times \frac{\left(\frac{1}{10}\right)}{\left(\frac{3}{10}\right)} = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

130.(A)

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx \quad \ldots(1)$$

$$= \int_0^{\frac{\pi}{2}} \log\left[\frac{4+3\sin\left(\frac{\pi}{2}-x\right)}{4+3\cos\left(\frac{\pi}{2}-x\right)}\right]dx = \int_0^{\frac{\pi}{2}} \log\left[\frac{4+3\cos x}{4+3\sin x}\right]dx \quad \ldots(2)$$

Eq. (1) + (2) gives,

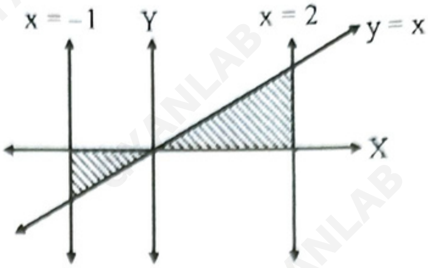
$$2I = \log\left[\frac{4+3\sin x}{4+3\cos x} \times \frac{4+3\cos x}{4+3\sin x}\right]dx = \int_0^{\frac{\pi}{2}} (\log 1)\,dx = 0$$

131.(C)

Required area is sahded.

$$A = \int_{-1}^{0} x\,dx + \int_{0}^{2} x\,dx$$

$$= \left|\left[\frac{x^2}{2}\right]_{-1}^{0}\right| + \left|\left[\frac{x^2}{2}\right]_{0}^{2}\right| = \frac{1}{2} + 2 = \frac{5}{2}$$



132.(C)

$$\text{Let } I = \int \frac{dx}{32 - 2x^2}$$

$$= \frac{1}{2}\int \frac{1}{16 - x^2}dx = \frac{1}{2}\int \frac{dx}{(4)^2 - (x)^2}$$

$$= \frac{1}{2}\left[\frac{1}{2(4)}\log\left|\frac{4+x}{4-x}\right|\right] + c = \frac{1}{16}[\log|4+x| - \log|4-x|] + c$$

Comparing with given data, we get $A = \frac{-1}{16}, B = \frac{1}{16}$

133.(B)

From given data, we write $A \equiv (0, b, c)$ and $B \equiv (a, 0, c)$

Equation of plane passing through A, B and O is

$$\begin{vmatrix} x-0 & y-0 & z-0 \\ 0-0 & b-0 & c-0 \\ a-0 & 0-0 & c-0 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x & y & z \\ 0 & b & c \\ a & 0 & c \end{vmatrix} = 0$$

$\therefore$ $x(bc) - y(-ac) + z(-ab) = 0 \Rightarrow bcx + acy - abz = 0$

Dividing both sides by abc, we get $\frac{x}{a} + \frac{y}{b} - \frac{z}{c} = 0$

134.(D)

$(\bar{a} + k\bar{b}) \cdot (\bar{a} - k\bar{b}) = 0$

$\therefore$ $|a|^2 - k^2|\bar{b}|^2 = 0$

$\therefore$ $(4)^2 - k^2(5)^2 = 0 \Rightarrow k = \pm\frac{4}{5}$

135.(D)

p : It is raining and q : It is pleasant.

So sumbolic form of 'It is neither raining nor pleasent' is $\sim p \vee \sim q$

136.(B)

$x^2 + y^2 - 4x - 10y + 25 = 0 \Rightarrow$ centre = (2, 5) and radius = $\sqrt{4 + 25 - 25} = 2$

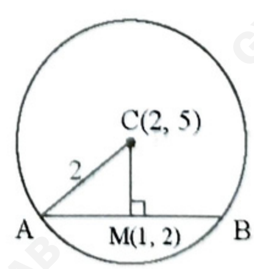
Refer figure

Let M(1, 2) be the mid point of chord

Slope of CM $= \frac{2-5}{1-2} = 3$

$\therefore$ slope of AB = $\frac{-1}{3}$

Equation of AB is $(y - 2) = \frac{-1}{3}(x - 1)$ i.e. $x + 3y = 7$



137.(B)

Let $I = \int \cos^3 x \cdot e^{\log(\sin x)} dx$

$= \int \cos^3 x \cdot \sin x \, dx$

Put $\cos x = t \Rightarrow -\sin x \, dx = dt$

$\therefore I = \int -t^3 dt = \frac{-t^4}{4} + t = \frac{-\cos^4 x}{4} + c$

138.(D)

We have $K + 2K + 3K + 4K + 5K + 6K = 1 \Rightarrow K = \frac{1}{21}$

$\therefore P(2 < x < 6) = P(x = 3) + P(x = 4) + P(x = 5)$

$= \frac{3}{21} + \frac{4}{21} + \frac{5}{21} = \frac{12}{21} = \frac{4}{7}$

139.(B)

$A = (-2, 2, 3); B = (3, 2, 2); C = (4, -3, 5)$ and $D = (7, -5, -1)$

$\overline{AB} = 5\hat{i} - \hat{k}$ and $\overline{CD} = 3\hat{i} - 2\hat{j} - 6\hat{k}$

Projection of $\overline{AB}$ on $\overline{CD} = \frac{\overline{AB} \cdot \overline{CD}}{|\overline{CD}|} = \frac{(5\hat{i} - \hat{k}) \cdot (3\hat{i} - 2\hat{j} - 6\hat{k})}{\sqrt{(3)^2 + (-2)^2 + (-6)^2}} = \frac{15+6}{7} = 3$

140.(A)

$(B^{-1}A^{-1})^{-1} = (A^{-1})^{-1}(B^{-1})^{-1} = AB$

$\therefore (B^{-1}A^{-1})^{-1} = \begin{bmatrix} 2 & -2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -3 & -2 \end{bmatrix}$

141.(D)

$\sin^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \frac{-\pi}{3} - \frac{\pi}{6} = \frac{-\pi}{2}$

142.(B)

$kx^2 + xy - y^2 = 0$

$\therefore k + \left(\frac{y}{x}\right) - \left(\frac{y}{x}\right)^2 = 0$ ...[Dividing both sides by x^2]

Slope of line is ± 1

$\therefore k + 1 - 1 = 0$ or $k - 1 - 1 = 0 \Rightarrow k = 0, 2$

143.(D)

$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & a \\ 2 & 4 & 7 \end{vmatrix} = (7 - 4a) - 2(7 - 2a) + 3(2) = -1$

Matrix B is inverse of matrix A. 'b' is element (1×3) in matrix B

Here element (3×1) in matrix A is 2.

Cofactor of 2 $= (-1)^{3+1} \begin{vmatrix} 2 & 3 \\ 1 & a \end{vmatrix} = 2a - 3$

Now $\frac{2a-3}{-1} = b \Rightarrow 2a + b = 3$ ...(1)

For element (1×2) in matrix A i.e. 2.

Now cofactor of 2 is $(-1)^{1+2}\begin{vmatrix}1 & a\\2 & 7\end{vmatrix} = -(7-2a)$ and $\frac{-(7-2a)}{-1} = 7-2a$

Here element (2×1) in matrix B is -3

$\therefore 7 - 2a = -3$ ...(2)

Solving eq. (1) and (2), we get $a = 5, b = -7$

144.(A)

Equation of parabola is $y^2 = 4a(x+a)$.

$\therefore \quad 2y\frac{dy}{dx} = 4a \quad \Rightarrow \quad a = \frac{y\frac{dy}{dx}}{2}$

Substituting value of 'a', we get

$$y^2 = 4\left(\frac{y\frac{dy}{dx}}{2}\right)\left[x + \left(\frac{y\frac{dy}{dx}}{2}\right)\right] = \left(2y\frac{dy}{dx}\right)\left(\frac{2x + y\frac{dy}{dx}}{2}\right)$$

$$2y^2 = \left(2y\frac{dy}{dx}\right)\left(2x + y\frac{dy}{dx}\right) = 4xy\frac{dy}{dx} + 2y^2\left(\frac{dy}{dx}\right)^2$$

$$\therefore \quad y = 2x\frac{dy}{dx} + y\left(\frac{dy}{dx}\right)^2$$

145.(C)

From 1 to 6, we have 1 and 4 as perfect squares.

Probability of getting perfect square in one throw of a die $= \frac{2}{6} = \frac{1}{3}$

$\therefore$ Probability of not getting perfect square in 4 throws of a die $= \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{16}{81}$

$\therefore$ Required probability $= 1 - \frac{16}{81} = \frac{65}{81}$

146.(C)

We have $|A\,(\text{adj } A)| = \begin{vmatrix}-10 & 0 & 0\\0 & -10 & 2\\0 & 0 & -10\end{vmatrix}$

$\therefore \quad |A|\,|\text{adj } A| = (-10)(100) = -1000$

$\therefore \quad |A|\,|A|^{3-1} = -1000 \Rightarrow |A|^3 = -1000 \Rightarrow |A| = -10$

147.(A)

$c(a \cos B - b \cos A)$

$= ac\left(\frac{c^2 + a^2 - b^2}{2ac}\right) - bc\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$

$= \frac{c^2 + a^2 - b^2}{2} - \frac{b^2 + c^2 - a^2}{2} = \frac{2a^2 - 2b^2}{2} = a^2 - b^2$

148.(B)

We have $2b^2 = a^2 + c^2$

$$\frac{\sin 3B}{\sin B} = \frac{3\sin B - 4\sin^3 B}{\sin B}$$

$$= 3 - 4\sin^2 B = 3 - 4(1 - \cos^2 B) \quad = 4\cos^2 B - 1$$

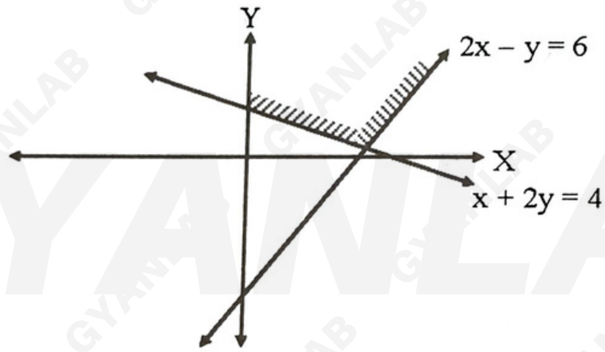
$$= 4\left[\frac{c^2 + a^2 - b^2}{2ac}\right]^2 - 1 \quad = 4\left[\frac{b^2}{2ac}\right]^2 - 1$$

$$= \left(\frac{2b^2}{2ac}\right)^2 - 1 = \left(\frac{a^2 + c^2}{2ac}\right)^2 - 1 = \left(\frac{a^2 + c^2}{2ac} + 1\right)\left(\frac{a^2 + c^2}{2ac} - 1\right) = \frac{(a+c)^2 (a-c)^2}{(2ac)^2}$$

$$= \left[\frac{(a+c)(a-c)}{2ac}\right]^2 \quad = \left(\frac{a^2 - c^2}{2ac}\right)^2$$

149.(B)

Refer Figure.



150.(C)

Let $\quad I = \int \frac{\cos x - \sin x}{8 - \sin 2x} dx$

$$= \int \frac{\cos x - \sin x}{9 - 1 - \sin 2x} dx \quad = \int \frac{\cos x - \sin x}{9 - (1 + \sin 2x)} dx$$

$$= \int \frac{\cos x - \sin x}{(3)^2 - (\sin x + \cos x)^2} dx$$

Put $\sin x + \cos x = t \Rightarrow (\cos x - \sin x)\, dx = dt$

$$\therefore \; I = \int \frac{dt}{(3)^2 - (t^2)} = \frac{1}{3(2)} \log\left|\frac{3+t}{3-t}\right| + c$$

$$= \frac{1}{6} \log\left|\frac{3 + \sin x + \cos x}{3 - \sin x - \cos x}\right| + c \Rightarrow p = 6$$